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Droplet entrainment correlation in annular two-phase flow

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Abstract—The droplet entrainment from a liquid film by gas flow is important to mass, momentum, and energy transfer in annular two-phase flow. The amount of entrainment can significantly affect occurrences of the dryout and post-dryout heat flux as well as the rewetting phenomena of a hot dry surface. In view of these, a correlation for the amount of entrained liquid in annular flow has been developed from a simple model and experimental data. There are basically two different regions of entrainment, namely, the entrance and quasi-equilibrium regions. The correlation for the equilibrium region is expressed in terms of the dimensionless gas flux, diameter, and total liquid Reynolds number. The entrance effect is taken into account by an exponential relaxation function. It has been shown that this new model can satisfactorily correlate wide ranges of experimental data for water. Furthermore, the necessary distance for the development of entrainment is identified. These correlations, therefore, can supply accurate information on entrainment which have not been available previously.

1. INTRODUCTION

THE DROPLET entrainment from a liquid film by a streaming gas flow is of considerable practical importance for heat and mass transfer processes in annular two-phase flow. The onset of droplet entrainment significantly alters the mechanisms of mass, momentum, and energy transfer between the film and gas core flow as well as the transfer between the two-phase mixture and the wall. In order to accurately predict a number of important physical phenomena in droplet–annular flow, an understanding of the mechanisms of the entrainment and correlation for the entrained fraction of liquid are essential. In particular, the dryout and post-dryout heat fluxes [1–5] and the effectiveness of the emergency core cooling systems in light water reactors [6–9] are significantly affected by the amount of liquid which can be entrained into gas core flow. However, the usefulness of an accurate correlation for the entrained fraction is not limited to these cases mentioned above. General thermohydraulic predictions in annular two-phase flow can be significantly improved by a reliable method of calculating the fraction of liquid flowing as droplets.

In spite of the importance of the entrained fraction for annular flow analyses, there have been no satisfactory correlations for the prediction of the amount of entrained liquid flow [10]. The lack of such a correlation has been the main difficulty of a detailed mechanistic modeling of various phenomena in annular two-phase flow. The purpose of the present study is to develop a reliable correlation for the entrained fraction of liquid. Furthermore, in view of the high probability of the occurrences of annular two-phase flow during LWR transients or accidents under various different conditions, a general correlation with wide ranges of applicability has been sought in the correlation development. In order to simplify the present analyses, the initial effort has been concentrated on the cases with low viscosity fluids such as water. Under this condition, the roll wave entrainment mechanism is the predominant mode of entrainment except at very low Reynolds numbers. Therefore, the present correlation is applicable to the entrainment due to the shearing-off of roll wave crest by streaming gas flow.

2. PREVIOUS WORK

When a gas phase is flowing over a liquid film, several different flow regimes are possible depending on the magnitude of the gas velocity [10–16]. For a very small gas velocity the interface is relatively stable, however, as the gas velocity increases the interfacial waves appear. The amplitude and irregularity of the waves become pronounced as the gas velocity is further increased. At sufficiently high gas flow, the capillary waves trasform into large amplitude roll waves [13, 14, 17–20]. Near the transition to the roll wave,

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NOMENCLATURE

C	droplet concentration in gas core [g cm^{-3}]
C_e	equilibrium droplet concentration in gas core
$\dot{D}$	droplet deposition rate
D	hydraulic diameter
D^*	dimensionless hydraulic diameter given by equation (17)
E	fraction of liquid flux flowing as droplet, j_{fe}/j_f
E_{th}	theoretical upper limit of E
E_∞	equilibrium value of fraction entrained, E
$\dot{E}$	entrainment rate
$f(\zeta)$	relaxation function for entrainment
F_σ	surface tension force
g	gravity
j_f	volumetric flux of total liquid (superficial velocity)
j_{fe}	volumetric flux of droplets
j_{ff}	volumetric flux of liquid film
j_g	volumetric flux of gas (superficial velocity)
j_g^*	dimensionless gas flux given by equation (15)
k	mass transfer coefficient
m	some power of droplet concentration
n	some power of Reynolds number
N_μ	viscosity number given by equation (8)
p	pressure
$(\mathrm{d}p/\mathrm{d}z)_k$	single phase pressure drop, if k-phase flows alone
R	entrainment parameter defined by equation (1)
Re_f	total liquid Reynolds number defined by equation (9)
v_f	liquid velocity
v_g	gas velocity
We	entrainment Weber number defined by equation (26)
We_c	critical Weber number of Dukler
W_f	total liquid mass flow rate
W_{fe}	droplet mass flow rate
X	Martinelli parameter
z	axial distance from inlet.

Greek symbols

δ	film thickness
ζ	dimensionless distance, equation (31)
η	equilibrium entrainment parameter, equation (25)
μ_f	viscosity of liquid
ρ_f	liquid density
ρ_g	gas density
ρ_{gc}	gas core density
$\Delta\rho$	density difference
σ	surface tension
τ_i	interfacial shear force.

Subscripts

f	liquid phase
g	gas phase
i	interface.

or at a still higher gas velocity, the onset of the entrainment occurs.

For a relatively high film Reynolds number ($Re_f > 160$), the mechanism of entrainment is basically due to the shearing-off of roll wave crests by a highly turbulent gas flow [12, 13]. However, at a lower Reynolds number, other entrainment mechanisms such as the wave-undercut become possible [10, 12, 14]. For a relatively inviscid fluid, the mechanism based on the roll wave is the predominant mode of the liquid entrainment into gas core flow under normal conditions.

Reviews of data and correlations for the onset of entrainment have been given by Zuber [21], Hewitt and Hall-Taylor [10], Wallis [20], Kutateladze [22], and Ishii and Grolmes [12]. Based on experimental data, van Rossum [14], Zhivaikin [23], Chien and Ibele [24], Zuber [21], Wallis [20], and Kutateladze [22] have obtained empirical correlations for the onset of droplet entrainment. The comparison of these correlations showed a number of conflicting results. Some of the correlations are dimensional and, therefore, their use is limited to narrow ranges of parameters. When these correlations are compared to data several important discrepancies have been observed [10, 12]. Reference [12] introduced a number of different regimes for entrainment and obtained a correlation from a simple modeling. This semi-empirical correlation appears to apply over wide ranges of conditions and to highly viscous fluids as well as liquid metals [12, 25].

A number of experiments have been carried out for the measurement of the entrained fraction. There are basically two different techniques [10] which can be used for this purpose. The first technique is based on local probes [3, 11, 26–30] such as the sampling or isokinetic probes which measures the axial liquid mass flux at the location of a probe. Normally, only the measurement at the centerline location is made and the uniformity of the flux is assumed. However, as has been noted by Hewitt and Hall-Taylor [10], the transverse distribution of the liquid mass flux may not be quite uniform [27, 31]. Theoretically it is possible to measure the flux distribution and then to integrate it over the gas core region. In practice, however, to do this becomes increasingly difficult at higher liquid flow, because a local probe starts to pick up liquid film flow near the wavy interface [30]. Therefore, the data based on a local probe always contains some uncertainties. The second and probably more accurate technique is based on the measurement of the liquid film flow by removing it completely from the test section [32–34].

The onset of entrainment and the amount of entrained liquid are sensitive to the geometry, particularly to the inlet conditions [28, 31]. The more abruptly liquid is introduced into a system, the higher is the amount of entrainment near the entrance section. At locations far from the inlet, the flow seems to reach quasi-equilibrium. However, it should be noted that in a low pressure system the pressure drop along a pipe causes the gas to expand and accelerate the flow. This higher gas velocity downstream results in further increases of the entrained fraction of liquid [3]. These two effects, namely, the entrance effect and gas expansion effect, should be carefully separated for a proper modeling of the entrainment phenomenon.

There are several correlations for the fraction of liquid flowing as entrained drops. Wicks and Dukler [26] correlated the droplet mass flow in terms of the Martinelli parameter, $X \equiv [(\mathrm{d}p/\mathrm{d}x)_f/(\mathrm{d}p/\mathrm{d}z)_g]^{1/2}$. Here $(\mathrm{d}p/\mathrm{d}z)_k$ is the single phase pressure drop which would exist if k-phase flowed alone. Then the entrainment parameter defined by

$$R \equiv \frac{We_c(j_f/j_g)}{(\mathrm{d}p/\mathrm{d}z)_g} W_{fe} \tag{1}$$

is correlated graphically to the Martinelli parameter X, where We_c and W_{fe} are the critical Weber number and droplet mass flow rate, respectively. The critical Weber number is 22 for very smooth entry of liquid and 13–16 for more abrupt entry. Although a reasonable agreement has been reported, this correlation is dimensional which limits the applicable ranges. Furthermore, the dependencies of the entrainment on various variables are hidden by the use of the Martinelli parameter.

The other correlations proposed by Minh and Huyghe [35], Paleev and Filipovich [33], and Wallis [36] are given in terms of the fraction of liquid entrained, E, and gas flux. Here E is defined as

$$E \equiv \frac{W_{fe}}{W_f} = \frac{j_{fe}}{j_f} \tag{2}$$

where W_{fe}, W_f, j_{fe}, and j_f are droplet mass flow rate, total liquid mass flow rate, droplet volumetric flux, and total liquid volumetric flux, respectively. Minh and Huyghe [35] plotted E against gas core momentum $\rho_{gc} j_g^2$ where ρ_{gc} is the homogeneous density of the gas core and given by $\rho_{gc} = \rho_g(1+\rho_f j_{fe}/\rho_g j_g)$. The result shows that the correlation depends on the fluid properties in an unspecified manner due to the use of the dimensional parameter.

This shortcoming was eliminated by Paleev and Filipovich [33] by introducing a dimensionless gas flux. Then, by fitting to data, the following empirical correlation has been obtained:

$$E = 0.015 + 0.44 \log\left[\frac{\rho_{gc}}{\rho_f}\left(\frac{\mu_f j_g}{\sigma}\right)^2 \times 10^4\right]. \tag{3}$$

This correlation showed fairly good agreement with a limited number of data, but the diameter effect and liquid Reynolds number dependence have been neglected. Later, Wallis [36] proposed to replace the liquid viscosity by the gas viscosity in Paleev and Filipovich's correlation. However, the dimensionless gas flux appearing in the correlations for the onset of entrainment [12, 22] as well as various data [27, 28] indicate that the viscosity dependence is more complicated than predicted by either correlation.

Apart from these overall correlations for the fraction of entrained liquid, a more mechanistic model has been proposed by Hutchinson and Whalley [37]. In their correlation method, the rates of entrainment and deposition of droplets have been introduced rather than using the equilibrium amount of entrainment used by others. The deposition rate $\dot{D}$ has been linearly related to the droplet concentration in the gas core by

$$\dot{D} = kC \tag{4}$$

whereas the rate of entrainment has been related to the equilibrium concentration C_e by

$$\dot{E} = kC_e \tag{5}$$

where k is the mass transfer coefficient. The equilibrium concentration has been correlated in a functional form

$$C_e = C_e\left(\frac{\tau_i \delta}{\sigma}\right) \tag{6}$$

where τ_i, δ, and σ are the interfacial shear, film thickness, and surface tension, respectively.

Although this correlation method is conceptually right and one step forward from the previous approach, there are some difficulties associated with it. First, in order to calculate the amount of entrainment two different correlations, i.e. equations (4) and (6) should be used. This requires that the deposition rate equation should be accurate simultaneously with the entrainment rate equation. In addition, the calculation involves integration of balance equations. The second difficulty is associated with the development of the correlation for the equilibrium concentration C_e. The data showed considerable scattering which can be up to more than one order of magnitude, particularly at small values of $\tau_i\delta/\sigma$. This suggests that a single dimensionless group used to correlate C_e is not sufficient. Furthermore, in many entrainment experiments such parameters as C_e and δ are not measured directly, thus some modeling to deduce data to useful forms has been necessary.

It can be said that existing correlations for entrainment fraction are not satisfactory. The entrainment rate and deposition rate are more mechanistic parameters representing the transfer of mass at the wavy interfaces than the entrainment fraction. However, it has been considered that the available data base is not sufficient to develop two rate correlations simultaneously. Therefore, as a first step, an attempt has been made to develop an accurate correlation for the entrainment fraction. This fraction represents the integral effects of the rate process, and therefore it represents more stable parameters to correlate than the rate equations.

3. ENTRAINMENT MODEL

3.1. *Theoretical limit of entrainment*

For low viscous fluids such as water, shearing-off of roll wave crests is the dominant mechanism of liquid entrainment into gas core flow in annular two-phase flow [12]. In view of this, the present model development is concentrated on the roll wave entrainment mechanism.

The onset of liquid entrainment has been studied [12] by considering a force balance at the crest of roll waves (Fig. 1). When the retaining force of surface tension is exceeded by the interfacial shear force exerted by the streaming gas flow, the droplet entrainment starts. This inception criterion is given for $Re_f > 2$ for vertical downflow and $Re_f > 160$ for vertical up or horizontal flow by

$$\frac{\mu_f j_g}{\sigma}\sqrt{\left(\frac{\rho_g}{\rho_f}\right)} \geqslant 11.78 N_\mu^{0.8} Re_f^{-1/3} \quad \text{for } N_\mu \leqslant 1/15 \tag{7}$$

where the viscosity number N_μ is given by

$$N_\mu = \mu_f/(\rho_f\sigma\sqrt{(\sigma/g\Delta\rho)})^{1/2} \tag{8}$$

and the liquid film Reynolds number by

$$Re_f = \frac{\rho_f j_f D}{\mu_f}. \tag{9}$$

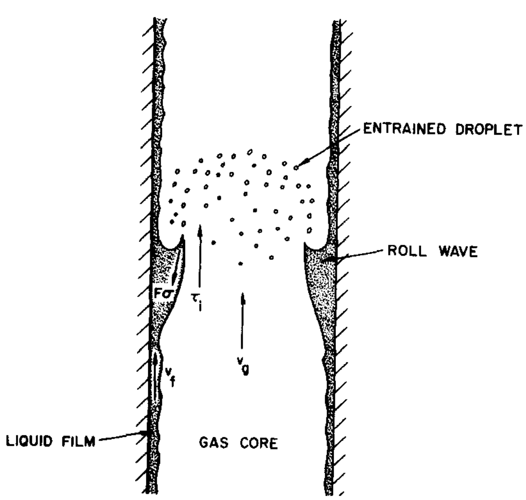

FIG. 1. Roll wave entrainment mechanism.

Furthermore, there is a rough turbulent regime where the critical gas velocity is independent of the liquid Reynolds number. This regime occurs for $Re_f > 1635$.

The above inception criterion is compared with experimental data in Fig. 2. Although there are considerable scattering of data around the criterion, mainly due to different measurement techniques used for different data and also due to the inclusion of data for highly viscous fluids [12], the overall trend is well predicted.

The theoretical limit for the amount of entrainment can be readily obtained from the above onset of entrainment criterion. If a system is operating above the lines representing the onset of the entrainment criterion in Figs. 2 and 3, then some liquid in the film should be entrained into the gas core flow. For the system to reach an equilibrium the liquid film Reynolds number should be reduced to a value which satisfies the inception criterion, if the effects of entrained drops on the criterion and the droplet deposition on the film can be neglected.

Thus, by assuming that all the excess liquid beyond the critical liquid flow will be entrained, one obtains from equation (7)

$$1-E_{th} = \frac{j_{ff}}{j_f} = \frac{\mu_f}{\rho_f D j_f}\left(11.78 N_\mu^{0.8}\frac{\sigma}{\mu_f j_g}\sqrt{\left(\frac{\rho_f}{\rho_g}\right)}\right)^3 \tag{10}$$

where E_{th} is the theoretical limit of the fraction entrained and j_{ff} is the liquid film volumetric flux. For a relatively inviscid fluid the following assumption can be made:

$$N_\mu^{0.8} = N_\mu N_\mu^{-0.2} \simeq 3N_\mu. \tag{11}$$

By substituting equations (8) and (11) into equation (10) one obtains

$$E_{th} = 1-\frac{1}{Re_f}\left[35\left(\frac{\sigma g\Delta\rho}{\rho_g^2}\right)^{1/4}\frac{1}{j_g}\right]^3. \tag{12}$$

This expression shows that E_{th} depends on the liquid Reynolds number and dimensionless gas flux given by

$$j_g^* \sim j_g/(\sigma g\Delta\rho/\rho_g^2)^{1/4}. \tag{13}$$

When this theoretical limit, E_{th}, was compared to experimental data, it was found that $E_{th} \gg E$, as expected. However, basic trends such as the dependencies on the gas flux and liquid Reynolds number are correctly predicted. This indicates that the same forces considered for the onset of entrainment also control the process of entrainment itself. From these observations one may expect that the entrained fraction E can be correlated by the dimensionless gas flux and liquid Reynolds number as $E = E(j_g^*, Re_f)$.

However, since there are many droplets flowing along the interface, a modification of the inertia of the gas core flow is necessary in order to take into account the droplet inertia. By assuming that this effect is not strongly coupled with the entrained fraction, a simple modification of the gas density has been adopted here. The dimensionless gas flux becomes

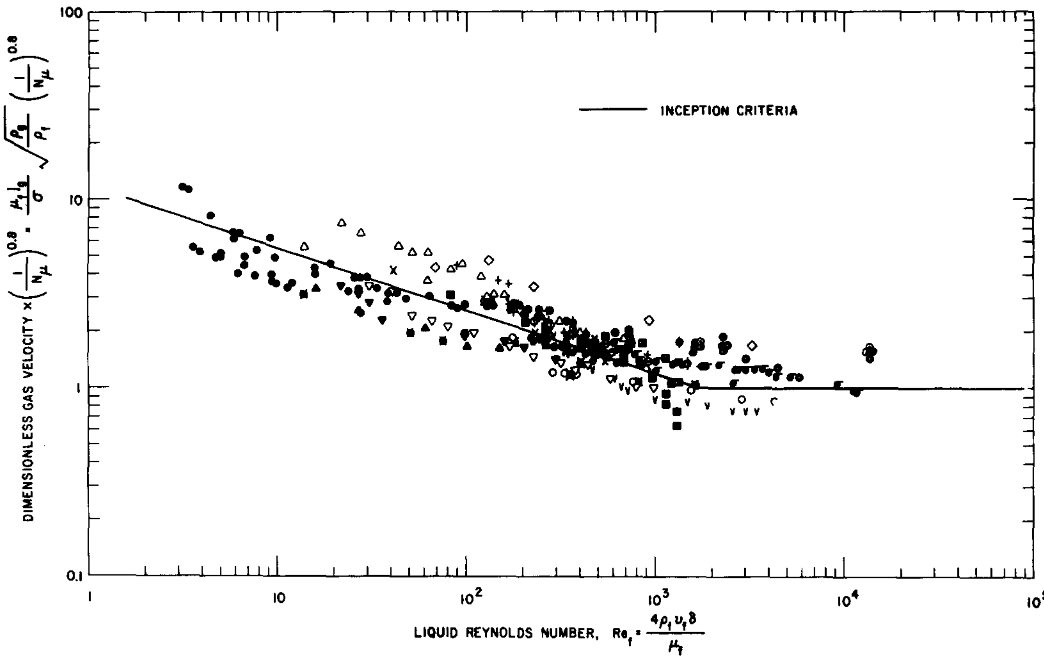

FIG. 2. Onset of entrainment criterion based on shearing-off of roll wave crests by Ishii and Grolmes [12].

$$j_g^* \sim j_g \Bigg/ \left[\frac{\sigma g \Delta\rho}{\rho_g^2 \left(\dfrac{\Delta\rho}{\rho_g}\right)^n}\right]^{1/4}. \qquad (14)$$

The experimental data of Steen and Wallis [28] plotted against this parameter at various liquid Reynolds numbers show that $n = 2/3$ is satisfactory (Figs. 4–6). Hence

$$j_g^* \equiv j_g \Bigg/ \left[\frac{\sigma g \Delta\rho}{\rho_g^2}\left(\frac{\rho_g}{\Delta\rho}\right)^{2/3}\right]^{1/4}. \qquad (15)$$

This form implies that the gas core inertia near the

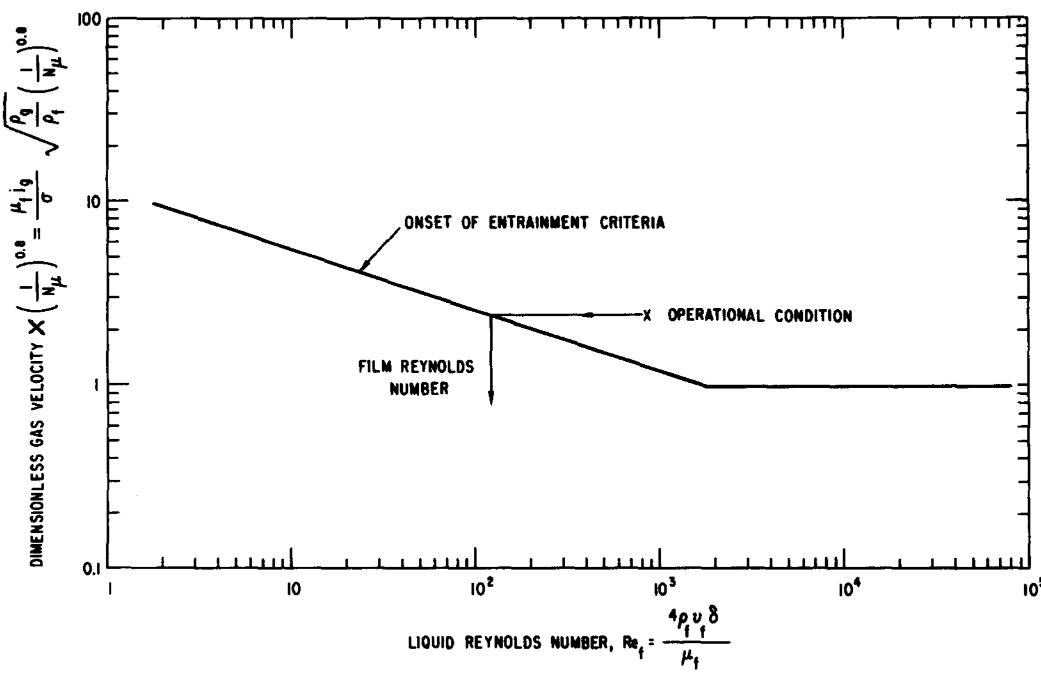

FIG. 3. Theoretical limit of entrainment from onset of entrainment criterion.

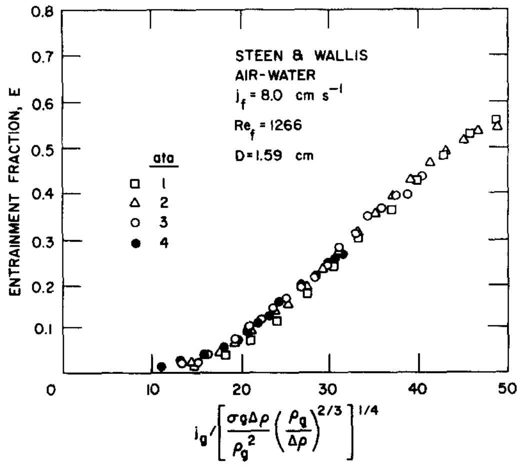

FIG. 4. Entrainment fraction vs dimensionless gas flux at $Re_f = 1266$ for data of Steen and Wallis [28].

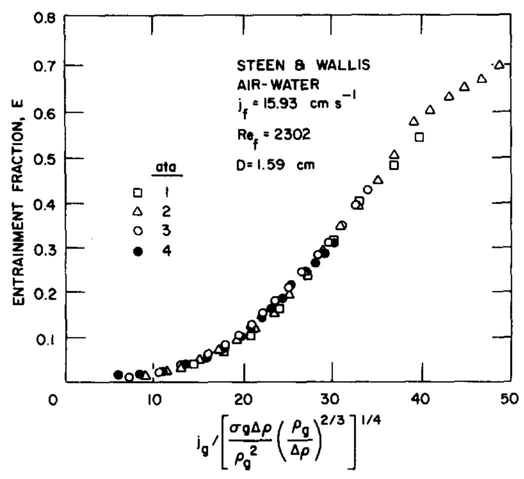

FIG. 5. Entrainment fraction vs dimensionless gas flux at $Re_f = 2302$ for data of Steen and Wallis [28].

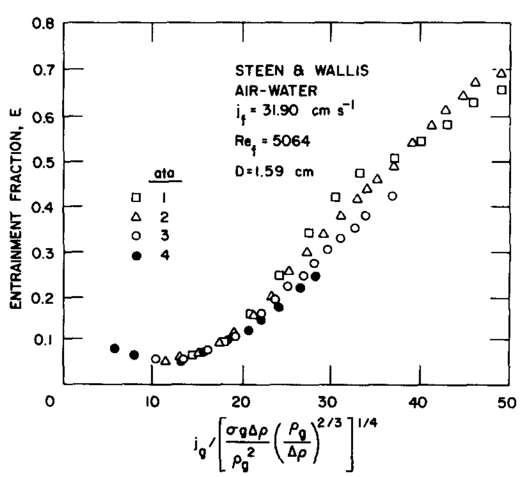

FIG. 6. Entrainment fraction vs dimensionless gas flux at $Re_f = 5064$ for data of Steen and Wallis [28].

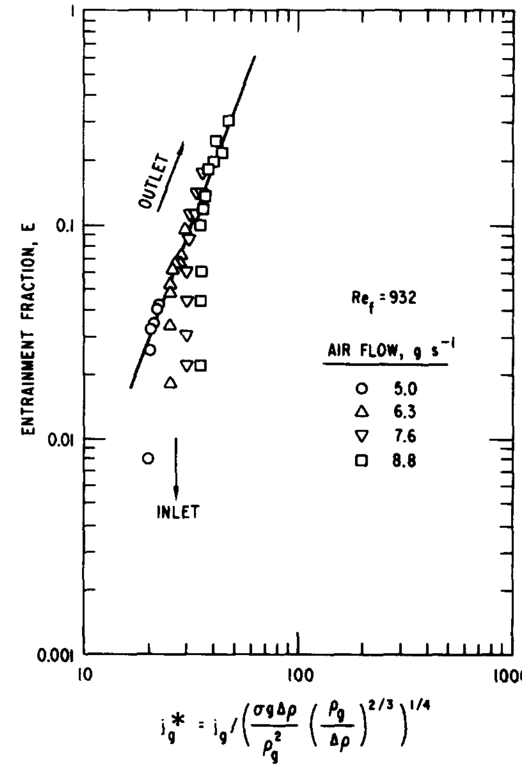

FIG. 7. Entrainment fraction vs dimensionless gas flux at $Re_f = 932$ for data of Cousins *et al.* [3] at $p = 0.276$ MPa (40 psi) and for $D = 0.95$ cm.

interface is increased by a factor of $(\Delta\rho/\rho_g)^{1/3}$ due to the existence of droplets.

3.2. *Fully developed entrainment*

The above analysis has indicated that the entrained fraction E depends on j_g^* and Re_f. Furthermore, from a simple consideration of droplet deposition it can be shown that E should also depend on the droplet concentration, hence on the diameter of a tube. This point will be discussed in detail later. For a fully developed flow, therefore

$$E = E_\infty(j_g^*, Re_f, D^*) \tag{16}$$

where D^* is the dimensionless hydraulic diameter scaled by the Taylor wavelength and given by

$$D^* \equiv D\sqrt{\left(\frac{g\Delta\rho}{\sigma}\right)}. \tag{17}$$

The expression for the theoretical limit of the entrained fraction, equation (12), implies that E should increase with increasing j_g^* and Re_f. Since the driving force is the gas core inertia, it is expected that E should be approximately proportional to j_g^{*2} except at very high values of j_g^*. Indeed, the plots of E vs j_g^* in Figs. 4–9 show that $E \sim j_g^{*2.5}$.

In addition, the data of Cousins *et al.* [3] clearly exhibit the entrance effect at locations near the entrance. When these data are plotted in terms of

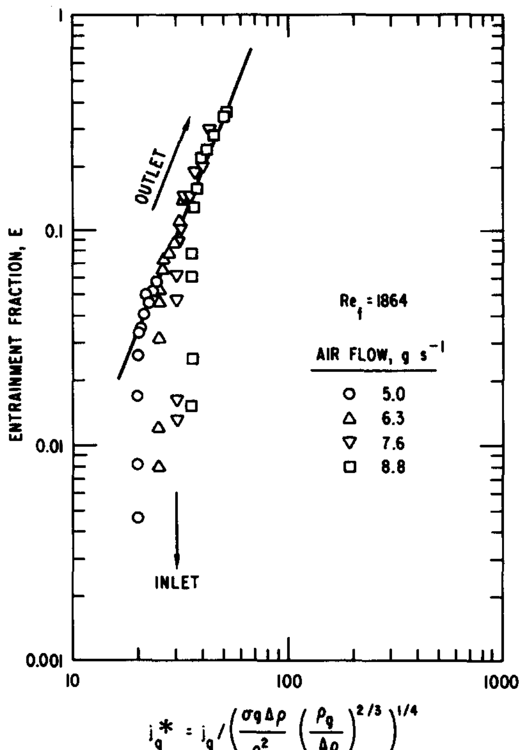

FIG. 8. Entrainment fraction vs dimensionless gas flux at $Re_f = 1864$ for data of Cousins *et al.* [3] at $p = 0.276$ MPa (40 psi) and for $D = 0.95$ cm.

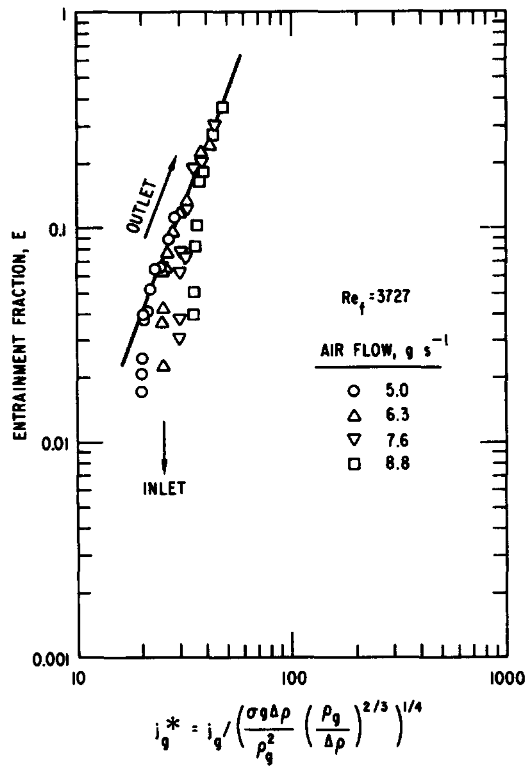

FIG. 9. Entrainment fraction vs dimensionless gas flux at $Re_f = 3727$ for data of Cousins *et al.* [3] at $p = 0.276$ MPa (40 psi) and for $D = 0.95$ cm.

$E/j_g^{*2.5}$ vs the axial distance, the value of $E/j_g^{*2.5}$ reaches an equilibrium value at locations far from the entrance. The asymptotic value of $E/j_g^{*2.5}$ depends on the liquid Reynolds number. This dependence of E_∞ on Re_f can be reasonably well correlated by

$$E_\infty \sim j_g^{*2} Re_f^{0.25} \quad (18)$$

as shown in Figs. 10–12. It is noted that this weaker dependence of E_∞ on Re_f than on j_g^* is also predicted by the expression for E_{th}.

Now the effect of the tube diameter or the flow area can be estimated by considering the process of droplet deposition. For systems having the same values of j_g^* and Re_f the entrained liquid flow rate is proportional to $E_\infty D$. This is because the total liquid flow rate is proportional to $j_f D^2$ whereas $j_f D = \text{const}$. Therefore, the droplet concentration is proportional to E_∞/D. However, the droplet deposition rate can be related to the concentration by

$$\dot{D} \sim (C)^m \sim \left(\frac{E_\infty}{D^*} \right)^m . \quad (19)$$

Therefore, in order to have the same deposition rate

$$E_\infty \sim D^* . \quad (20)$$

However, the various experimental data used in this study indicated that $E_\infty \sim D^{*1.25}$. Thus, by using this modified value, one obtains

$$E_\infty \sim j_g^{*2.5} D^{*1.25} Re_f^{0.25} . \quad (21)$$

Finally, there are two limiting conditions which a correlation for E_∞ should satisfy. They are

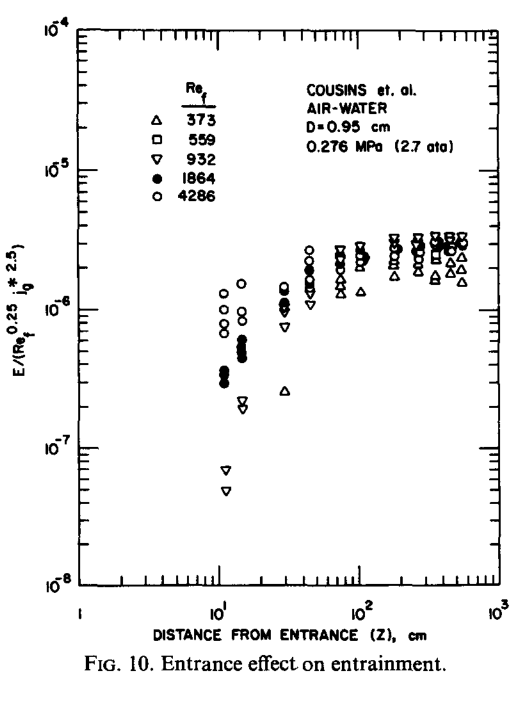

FIG. 10. Entrance effect on entrainment.

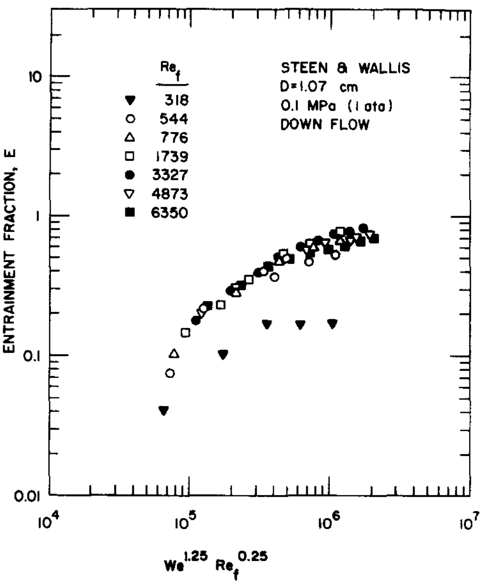

Fig. 11. Effect of Reynolds number on entrainment from data of Steen and Wallis for $D = 1.07$ cm.

$$E_\infty \ll 1 \text{ for } j_g^* \leqslant j_{gc}^*$$

$$E_\infty \to 1 \text{ for } j_g^* \to \infty \tag{22}$$

where j_{gc}^* is the dimensionless critical gas flux for the onset of entrainment corresponding to equation (7). The first limit imposes a condition that the entrained fraction of liquid should be practically zero at gas fluxes below the critical value. The second condition implies that the maximum possible value of E_∞ is 1. The expression given by the following equation:

$$E_\infty = \tanh(7.25 \times 10^{-7} j_g^{*2.5} D^{*1.25} Re_f^{0.25}) \tag{23}$$

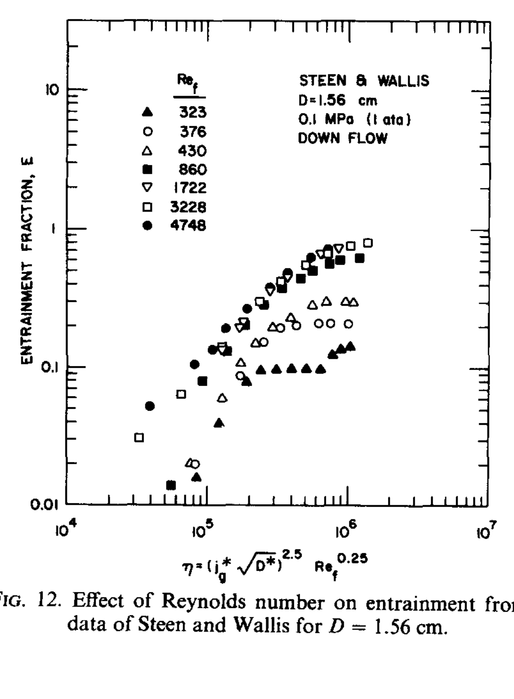

Fig. 12. Effect of Reynolds number on entrainment from data of Steen and Wallis for $D = 1.56$ cm.

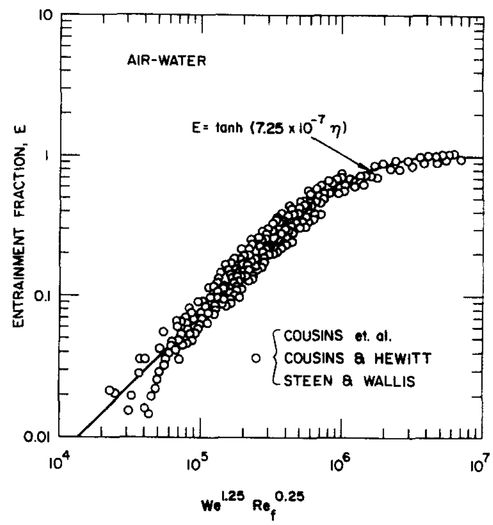

Fig. 13. Comparison of equilibrium entrainment correlation to various data.

can fit to a large number of data taken under various different conditions and also satisfies the limit imposed above. The comparison of the data of Cousins *et al.* [3], Cousins and Hewitt [34], and Steen and Wallis [28] to the above correlation is shown in Fig. 13.

It is interesting to note that the correlating parameter given by

$$\eta = j_g^{*2.5} D^{*1.25} Re_f^{0.25} \tag{24}$$

can be expressed in a different form. By substituting the definitions for j_g^*, D^*, and Re_f, η becomes

$$\eta = \left[\frac{\rho_g j_g^2 D}{\sigma}\left(\frac{\Delta\rho}{\rho_g}\right)^{1/3}\right]^{1.25}\left(\frac{\rho_f j_f D}{\mu_f}\right)^{0.25}. \tag{25}$$

Hence an effective Weber number for entrainment is defined by

$$We = \frac{\rho_g j_g^2 D}{\sigma}\left(\frac{\Delta\rho}{\rho_g}\right)^{1/3}. \tag{26}$$

Then

$$\eta = We^{1.25} Re_f^{0.25}. \tag{27}$$

The important point here is that if the Weber number is used, the significant length scale is D. Then the Taylor wavelength $\sqrt{(\sigma/\Delta\rho g)}$ which is the standard length scale for interfacial phenomena does not appear in the correlation. Thus in terms of the entrainment Weber number and liquid Reynolds number the equilibrium entrainment correlation becomes

$$E_\infty = \tanh(7.25 \times 10^{-7} We^{1.25} Re_f^{0.25}). \tag{28}$$

The entrainment fraction correlation given by equation (28) is for the quasi-equilibrium condition. Therefore, it applies to a region far away from the entrance. In the following section the effect of the development of the flow and the length of the entrance region are studied.

3.3. *Effect of entrance region on entrainment*

A number of data indicate that there is an entrance effect on the droplet entrainment as shown in Figs. 10–12. For a system with smooth injection of liquid, entrainment is very small near the inlet due to the insufficient interaction between the gas core flow and the liquid film. However, the amount of entrained liquid continuously increases with the axial distance as can be seen from Fig. 10. This entrance effect was studied experimentally by Gill and Hewitt [31] and Cousins *et al* [3]. The amount of entrainment was found to be very sensitive to the inlet conditions. In some cases, the effect of liquid injection methods can persist to a great distance from the inlet. In the present study, only the case with relatively smooth injection of liquid as a film is considered.

In studying the entrance effect, the gas expansion effect due to axial pressure drop should be carefully distinguished from the entrance effect itself. If these effects are not separated the entrainment in a low pressure system seems never to reach a quasi-equilibrium condition and the entrained fraction continuously increases downstream due to the gas expansion. Therefore, it is very important to use a local gas velocity or volumetric flux based on a local pressure for this analysis. The data of Cousins *et al.* [3] clearly show that there is an entrance region and gas expansion region. Furthermore, the effect of the gas expansion can be very strong as indicated by the correlation for the fraction entrained, i.e. equation (21) or equation (23). Note that E is approximately proportional to $j_g^{2.5}$.

In assessing the effect of the entrance region, a proper length scale should be considered to take account of a flow development due to interfacial transport of mass and momentum. For this purpose some estimate of the rate process of entrainment becomes necessary. Since entrainment is caused by shearing-off of wave crests of a liquid film, it is expected that the rate should be proportional to an interfacial drag force. Therefore, $\dot{E} \sim \rho_g j_g^2/2$ or j_g^{*2}. This force is also proportional to the wave amplitude. Without going into a detailed non-linear analysis to solve for the wave amplitude it is estimated that the amplitude is proportional to some power to the liquid flow or Reynolds number, Re_f^n. Here n should be close to 1. This Reynolds number dependence of the entrance effect can be clearly seen in Figs. 10–12. Thus

$$\dot{E} \sim j_g^{*2}\, Re_f^n. \tag{29}$$

Using the above expression for the entrainment rate the development of droplet entrainment can be scaled by

$$\frac{E}{E_\infty} \sim \frac{\int \dot{E}\,\mathrm{d}z}{E_\infty} \sim \frac{j_g^{*2}\, Re_f^n\, z}{j_g^{*2.5}\, Re_f^{0.25}\, D^{1.25}}. \tag{30}$$

Therefore, a length scale ζ can be given approximately by

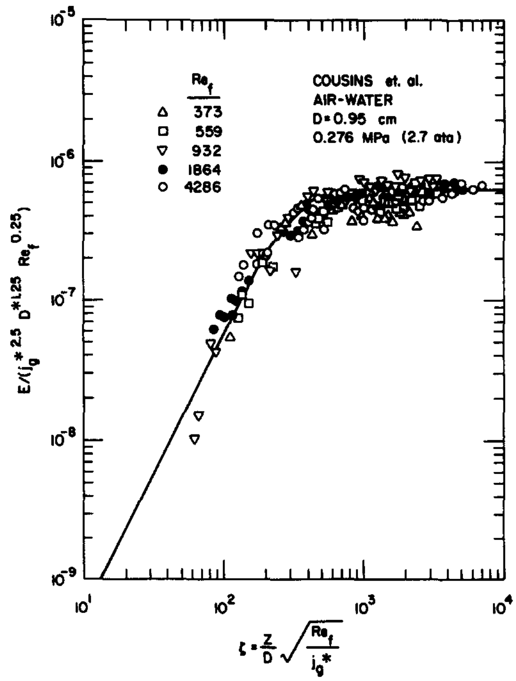

FIG. 14. Effect of entrance region on entrainment, and comparison of correlation to data.

$$\zeta \sim \frac{z}{D}\frac{Re_f^{n-0.25}}{j_g^{*0.5}} \tag{31}$$

where an additional small dependence on D^* has been neglected. It is shown in Fig. 14 that the value of $n = 0.75$ can satisfactorily correlate the entrance effect on entrainment.

Then, by defining ζ as

$$\zeta \equiv \frac{z}{D}\sqrt{\left(\frac{Re_f}{j_g^*}\right)} \tag{32}$$

the correlation for the entrained fraction in an entrance region is given by

$$E = (1 - \mathrm{e}^{-10^{-5}\zeta^2})E_\infty \tag{33}$$

where E_∞ is the quasi-equilibrium value of the fraction entrained and given by equation (23). In this case, the function is given by

$$f(\zeta) = (1 - \mathrm{e}^{-10^{-5}\zeta^2}) \tag{34}$$

that expresses the development of droplet entrainment along the axial distance.

This exponential relaxation function can fit to data in the entrance region reasonably well as shown in Fig. 14. Furthermore, the data indicate that the entrance region is given approximately by

$$0 < \zeta < 600. \tag{35}$$

Thus, the entrainment reaches a quasi-equilibrium value for

$$z \gtrsim 600 D \sqrt{\left(\frac{j_g^*}{Re_f}\right)}. \tag{36}$$

For example, a typical case of $Re_f = 2000$, $j_g^* = 40$, and $D = 1$ cm has the entrance length of 85 cm from expression (35). This shows that in many practical cases the effect of the entrance region may not be negligible, particularly for a short tube.

The above expression also indicates that the entrance length increases with the increasing gas flux or hydraulic diameter. This effect of the gas flux may be explained by the reduced residence time for gas flow at higher gas velocities. On the other hand, the entrance length decreases with the increasing liquid Reynolds number. This Reynolds number dependence can be explained by an increased entrainment rate for higher Re_f due to the larger wave amplitude. In contrast, the hydrodynamic entrance effect in a single phase flow is scaled by $z/(D\,Re_f)$ which is based on the boundary layer development. Therefore, the effect of the Reynolds number on the entrance length is completely opposite in the film entrainment phenomenon.

4. SUMMARY AND CONCLUSIONS

A correlation for the amount of entrained liquid in annular two-phase flow has been developed from a simple model and experimental data. The fraction of liquid flux flowing as droplets, E, is correlated in terms of three dimensionless groups, namely, dimensionless gas flux, total liquid Reynolds number, and dimensionless diameter.

The entrained fraction reaches a quasi-equilibrium value E_∞, at points far removed from the tube entrance where the entrainment and deposition processes attain an equilibrium condition. The distance necessary to reach this condition is given approximately by $Z = 600D\sqrt{(j_g^*/Re_f)}$ for cases with smooth liquid injection as a film. At this entrance length the entrainment has reached within about 2% of its ultimate value. Then for the region $z > 600D\sqrt{(j_g^*/Re_f)}$ the correlation is given by equation (23) or in terms of the special entrainment Weber number by equation (28).

Because of the nature of the roll wave entrainment, the above correlation may be limited to $Re_f > 2$ for vertical down flow and $Re_f > 160$ for vertical up or horizontal flow. Furthermore, from the restriction on the viscosity number this correlation may not work for a highly viscous fluid where $N_\mu \gg 0.1$.

This correlation has been compared to many experimental data for air–water systems in the ranges of $1 < p < 4$ atm, $0.95 < D < 3.2$ cm, $370 < Re_f < 6400$, and $j_g < 100$ m s^{-1}, and the result has been shown to be satisfactory. The various parametric dependencies have been explained in terms of physical mechanisms and information obtained from the onset of entrainment criterion developed previously.

Some experimental data indicated the strong entrance effect as well as the gas expansion effect due to the axial pressure drop in a low pressure system. For the correlation development it was essential to use a local gas velocity or volumetric flux based on a local pressure in evaluating data. By separating these two effects an additional correlation for the entrance effect on entrainment have been developed for the case of the smooth liquid injection as film. The correlation takes a typical form of an exponential relaxation, and it essentially reaches the quasi-equilibrium value given by E_∞ for large values of z. A number of data from the entrance region have been successfully correlated by this expression. This inclusion of entrance effect in the correlation is a significant improvement over the conventional correlations.

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REFERENCES

1. J. G. Collier, Burnout in liquid cooled reactors, *Nucl. Pwr* **5**, 61 (1961).
2. V. I. Petrovichev, L. S. Kokorev, A. Ya. Didenko and G. P. Dubvrovskiy, Droplet entrainment in boiling of thin liquid film, *Heat Transfer—Sov. Res.* **3**, 19 (1971).
3. L. B. Cousins, W. H. Denton and G. F. Hewitt, Liquid mass transfer in annular two-phase flow, *Symp. on Two-phase Flow*, Exeter, U.K., Vol. 1, Paper C4 (1965).
4. L. S. Tong, *Boiling Heat Transfer and Two-phase Flow*. Krieger, New York (1975).
5. Y. Y. Hsu and R. W. Graham, *Transport Process in Boiling and Two-phase Systems*. Hemisphere, Washington, DC (1976).
6. A. Yamanouchi, Effect of core spray cooling at stationary state after loss of coolant accident, *J. Nucl. Sci. Technol.* **5**(9), 498 (1968).
7. R. Semeria and B. Martinet, Calefaction spots on a heating wall; temperature distribution and resorption, *Symp. on Boiling Heat Transfer in Steam Generating Unit and Heat Exchangers, Proc. Inst. Mech. Engrs*, Vol. 180, pp. 192–205 (1965).
8. R. Duffey, The physics of rewetting in water reactor emergency core cooling, *Nucl. Engng Des.* **25**, 379 (1973).
9. E. Elias and G. Yadigaroglu, The reflooding phase of the LOCA state of the art II, rewetting and liquid entrainment, *Proc. NATO Advanced Study Inst., Two-phase Flows and Heat Transfer*, Vol. 2, pp. 1009–1040 (1976).
10. G. F. Hewitt and N. S. Hall-Taylor, *Annular Two-phase Flow*, pp. 136–162. Pergamon Press, New York (1970).
11. G. B. Wallis, The onset of droplet entrainment in annular gas–liquid flow, 62 GL 127, General Electric (1962).
12. M. Ishii and M. A. Grolmes, Inception criteria for droplet entrainment in two-phase concurrent film flow, *A.I.Ch.E. Jl* **21**, 308 (1975).
13. R. S. Brodkey, *The Phenomena of Fluid Motions*, pp. 561–565. Addison-Wesley, Reading, Massachusetts (1967).
14. J. J. van Rossum, Experimental investigation of horizontal liquid films, *Chem. Engng Sci.* **11**, 35 (1959).
15. V. G. Levich, *Physicochemical Hydrodynamics*, pp. 395–692. Prentice-Hall, Englewood Cliffs, New Jersey (1962).
16. H. Lamb, *Hydrodynamics*, pp. 456–464. Dover, New York (1945).
17. T. J. Hanratty and J. M. Engen, Interaction between a turbulent air stream and a moving water surface, *A.I.Ch.E. Jl* **3**, 299 (1957).
18. T. J. Hanratty and A. Hershman, Initiation of roll waves, *A.I.Ch.E. Jl* **7**, 488 (1961).

19. H. S. Chung and W. Murgatroyd, Studies of the mechanism of roll wave formation on thin liquid films, *Symp. on Two-phase Flow*, Exeter, U.K., Vol. 2, Paper A2 (1965).
20. G. B. Wallis, *One Dimensional Two-phase Flow*, pp. 345–351. McGraw-Hill, New York (1969).
21. N. Zuber, On the atomization and entrainment of liquid films in shear flow, 62 GL 153, General Electric (1962).
22. S. S. Kutateladze, Elements of the hydrodynamics of gas liquid system, *Fluid Mech.—Sov. Res.* **1**, 29 (1972).
23. L. Ya. Zhivaikin, Liquid film thickness in film-type units, *Int. Chem. Engng* **2**, 337 (1962).
24. S. F. Chien and W. Ibele, Pressure drop and liquid film thickness of two-phase annular and annular–mist flows, ASME Paper 62-WA-170 (1960).
25. M. Ishii and M. A. Grolmes, Prediction of onset of entrainment for liquid metals, *Trans. Am. Nucl. Soc.* **21**, 325 (1975).
26. M. Wicks and A. E. Dukler, Entrainment and pressure drop in concurrent gas–liquid flow: air–water in horizontal flow, *A.I.Ch.E. Jl* **6**, 463 (1960).
27. P. G. Magiros and A. E. Dukler, Entrainment and pressure drop in concurrent gas–liquid flow, *Dev. Mech.* **1**, 532 (1961).
28. D. A. Steen and G. B. Wallis, The transition from annular to annular–mist concurrent two-phase down flow, AEC Report NYO-3114-2 (1964).
29. R. M. Yablonik and V. A. Khaimov, Determination of the velocity of inception of droplet entrainment in two-phase flow, *Fluid Mech.—Sov. Res.* **1**, 130 (1972).
30. T. Ueda and T. Tanaka, Studies of liquid film flow in two-phase annular and annular–mist flow regions, parts 1 and 2, *Trans. J.S.M.E.* **39**, 2842 (1973).
31. L. E. Gill and G. F. Hewitt, Sampling probe studies of the gas core in annular two-phase flow—III. Distribution of velocity and droplet flow rate after injection through an axial jet, *Chem. Engng Sci.* **23**, 677 (1968).
32. I. I. Paleev and F. A. Agafonova, Heat transfer between a hot surface and a gas flow entraining drops of evaporating liquid, *Proc. 1st All-Union Heat and Mass Transfer Conf.*, Minsk, Vol. 2, p. 260 (1962).
33. I. I. Paleev and B. S. Filipovich, Phenomena of liquid transfer in two-phase dispersed annular flow, *Int. J. Heat Mass Transfer* **9**, 1089 (1966).
34. L. B. Cousins and G. F. Hewitt, Liquid phase mass transfer in annular two-phase flow: droplet deposition and liquid entrainment, UKAEA Report AERE-R5657 (1968).
35. T. Q. Minh and J. D. Huyghe, Some hydrodynamical aspects of annular dispersed flow: entrainment and film thickness, *Symp. on Two-phase Flow*, Exeter, U.K., Vol. 1, Paper C2 (1965).
36. G. B. Wallis, Phenomena of liquid transfer in two-phase dispersed annular flow, *Int. J. Heat Mass Transfer* **11**, 783 (1968).
37. P. Hutchinson and P. B. Whalley, A possible characterization of entrainment in annular flow, *Chem. Engng Sci.* **28**, 974 (1973).

FORMULE D'ENTRAINEMENT DE GOUTTELETTES DANS UN ECOULEMENT ANNULAIRE DIPHASIQUE

Résumé—L'entrainement de gouttelettes à partir d'un film liquide par un écoulement de gaz est important pour le transfert de masse, de quantité de mouvement et d'énergie dans l'écoulement annulaire diphasique. Le taux d'entrainement peut affecter significativement l'assèchement et le flux thermique ultérieur aussi bien que les phénomènes de remouillage d'une surface chaude et sèche. Une formule pour la quantité de liquide entraînée dans l'écoulement annulaire a été développée à partir d'un modèle simple et de données expérimentales. Il y a deux régions différentes d'entraînement, celle d'entrée et celle de quasi-équilibre. La formule pour la région d'équilibre est exprimée en fonction du flux adimensionnel de gaz, du diamètre et du nombre de Reynolds total de liquide. L'effet d'entrée est pris en compte par une fonction de relaxation exponentielle. On montre que ce nouveau modèle peut unifier de larges domaines de données expérimentales sur l'eau. On identifie la distance nécessaire au développement de l'entrainement. Ces formules peuvent fournir une information précise sur l'entrainement qui n'existait pas auparavant.

EINE KORRELATION FÜR DAS MITREISSEN VON TRÖPFCHEN IN EINER ZWEIPHASIGEN RINGSTRÖMUNG

Zusammenfassung—Das Mitreißen von Tropfen aus einem Flüssigkeitsfilm durch einem überlagerten Gasstrom ist wichtig für den Stoff-, Impuls- und Energietransport in einer zweiphasigen Ringströmung. Die Menge der mitgerissenen Flüssigkeit kann wesentlichen Einfluß auf die Vorgänge beim Austrocknen der Heizfläche und auf die Wärmestromdichte im 'post-dryout'-Bereich nehmen, ebenso auf die Wiederbenetzung trockener, heißer Oberflächen. Vor diesem Hintergrund wird aufgrund eines einfachen Modells und mit Hilfe von Versuchsdaten eine Korrelation für die Menge der mitgerissenen Flüssigkeit entwickelt. Zwei grundlegend unterschiedliche Bereiche sind zu betrachten: der Eintrittsbereich und der Bereich mit einem Quasi-Gleichgewicht. Die Korrelation für den Gleichgewichts-Bereich enthält die dimensionslose Gasstromdichte, den Durchmesser und die Gesamt-Reynolds-Zahl für die Flüssigkeit. Der Eintrittseffekt wird mit einer exponentiellen Relaxationsfunktion berücksichtigt. Es zeigt sich, daß dieses neue Modell in der Lage ist, Versuchsdaten für Wasser in einem weiten Parameterbereich zufriedenstellend wiederzugeben. Außerdem wird die notwendige Lauflänge für die Ausbildung des Mitreißens festgestellt. Damit können diese Korrelationen genaue Informationen über das Mitreißen von Flüssigkeit bereitstellen, die bisher nicht verfügbar waren.

УНОС КАПЕЛЬ В КОЛЬЦЕВОМ ДВУХФАЗНОМ ПОТОКЕ

Аннотация—Унос капель из пленки жидкости потоком газа является важным фактором переноса массы, импульса и энергии при кольцевом двухфазном течении. Количество уносимой жидкости может оказывать сильное влияние на обстоятельства осушения, тепловой поток после осушения, а также на явления вторичного смачивания горячей сухой поверхности. С учетом вышеизложенного на основе простой модели и экспериментальных данных получено соотношение для определения количества уносимой кольцевым потоком жидкости. Существуют две принципиально различные области уноса, а именно, входная и квазиравновесная. Соотношение для равновесной области выражено через безразмерный поток газа, диаметр и результирующее число Рейнольдса для жидкости. Влияние входа учитывается экспоненциальной функцией релаксации. Показано, что с помощью новой модели можно удовлетворительно обобщать экспериментальные данные для воды, изменяющиеся в широких пределах. Установлен размер, необходимый для развития уноса. Полученные зависимости позволяют иметь более точную информацию об уносе, которая не была доступна ранее.